

Hidden Quantum Critical Point in a Ferromagnetic Superconductor

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We consider a coexistence phase of both Ferromagnetism and superconductivity and solve the self-consistent mean-field equations at zero temperature. The superconducting gap is shown to vanish at the Stoner point whereas the magnetization doesn't. This indicates that the para-Ferro quantum critical point becomes a hidden critical point. The effective mass in such a phase gets enhanced whereas the spin wave stiffness is reduced as compared to the pure FM phase. The spin wave stiffness remains finite even at the para-Ferro quantum critical point.

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The study of itinerant Ferromagnetic materials are becoming more and more important due to the role of strong electronic correlations and the appearance of many exotic phases such as non-Fermi liquid (NFL), superconductivity (SC) etc. near the para-Ferro quantum critical point (QCP). In earlier times, Ferromagnetism (FM) was believed to suppress SC but the recent discovery of SC [1] below 1 K in the pressure range of 1 to 1.6 GPa in a high purity single crystal of UGe_2 has ruled out the above possibility. This rather suggests that FM and SC could be cooperative. Also, there are other materials such as ZrZn_2 [2] and URhGe [3], where the coexistence of FM and SC has been found. SC phase in all the above mentioned materials is completely covered within the FM phase and disappears in the paramagnetic (PM) region.

The standard way to look for a coexistence phase of FM and SC theoretically is to introduce two kinds of fermions. FM could be caused by local f-electrons whereas SC, by itinerant ones. But in the above materials such as, UGe_2 and URhGe , both the roles were played by the same Uranium 5-f electrons which are itinerant and strongly correlated. Thus, it would be interesting to study microscopically a model where the coexistence of both FM and SC can be described by only one kind of electrons. Such a model study has recently been initiated by Karchev et. al. [4]. Of course, this model is confined to singlet SC which is unlikely to occur inside a FM.

In this letter, we study the coexistence phase of both FM and SC and look for the consequences. We solve the zero temperature self-consistent mean-field equations. It is shown that the SC gap vanishes at the para-Ferro QCP in such a model whereas the magnetization doesn't. This suggests that the SC pairing induces a small but finite magnetization which doesn't vanish even at the Stoner threshold. This is an indication of the para-Ferro QCP becoming a hidden QCP. We also computed the effective mass as well as the spin wave stiffness in the coexistence phase. The later eventhough reduced, is nonzero at the para-Ferro QCP.

We presume that the relevant magnetic behaviour of the system is adequately described by Stoner RPA-mean-field theory [5] and do not question on its stability. This

can be obtained from Hubbard Hamiltonian. In order to get SC pairing, we add a reduced BCS Hamiltonian to it. We would like to mention here that the pairing Hamiltonian is not due to the FM spin fluctuation rather it may be due to some other means. Thus, to describe a coexistence phase of both FM and SC, one can start with a minimal effective Hamiltonian,

$$H = \sum_{k,\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - V \sum_{k,k'} c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger c_{-k'\downarrow} c_{k'\uparrow}, \quad (1)$$

where U and V are respectively the on-site Hubbard interaction energy and the reduced BCS pairing energy. $n_{k\sigma} = c_{k\sigma}^\dagger c_{k\sigma}$ is the electron density and $c_{k\sigma}^\dagger (c_{k\sigma})$ are the standard electron creation (annihilation) operator with wave vector k and spin projection σ . In order to obtain a coexistence phase of both FM and SC, we can perform a mean-field theory by defining the averages, $2\Delta_F = U(\langle n_\downarrow \rangle - \langle n_\uparrow \rangle)$, and $\Delta = V \sum_k \langle c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger \rangle$. Here $2\Delta_F$ and Δ are respectively the FM and the SC order parameter in the coexistence phase. At this stage, one can diagonalize the above Hamiltonian through the standard Bogoliubov transformation and the new energy dispersions are obtained as,

$$E_k^\alpha = \Delta_F + \sqrt{(\epsilon_k - \mu)^2 + |\Delta|^2}, \quad (2)$$

$$E_k^\beta = \Delta_F - \sqrt{(\epsilon_k - \mu)^2 + |\Delta|^2}. \quad (3)$$

The subscript α and β above denote the two different Bogoliubov Fermions in the coexistence phase. The self-consistent mean-field equations are derived as,

$$2\Delta_F = \lambda \int d\epsilon (1 - n_k^\beta - n_k^\alpha), \quad (4)$$

$$|\Delta| = g \int d\epsilon \frac{|\Delta|}{2E} (n_k^\beta - n_k^\alpha), \quad (5)$$

where $n_k^{\beta,\alpha}$ are the momentum distribution for the corresponding Bogoliubov Fermions and $\lambda = U\rho(0)$, $g =$

$V\rho(0)$, $E_k = \sqrt{(\epsilon_k - \mu)^2 + |\Delta|^2}$, $\rho(0)$, being the density of state at the Fermi level in the PM phase.

It is obvious from equations (2) and (3) that for $\Delta_F > 0$, $E_k^\alpha > 0$ for all k and thus, at zero temperature, $n_k^\alpha = 0$. On the other hand, for E_k^β , there are two possibilities, (i) $E_k^\beta > 0$ and (ii) $E_k^\beta < 0$. In case (ii) $n_k^\beta = 1$ for all k . Substituting this in equation (4) yields $\Delta_F = 0$. Thus, the only solution for equation (4) which allows nonzero Δ_F in order to get a coexistence phase is $E_k^\beta > 0$. The dispersion of the β -Fermion becomes positive only in the energy interval $\epsilon_F^- < \epsilon_k < \epsilon_F^+$, where ϵ_F^\pm are the solutions of the equation $E_k^\beta = 0$, which is given as, $\epsilon_F^\pm = \epsilon_F \pm \sqrt{\Delta_F^2 - |\Delta|^2}$. It should be noted here that ϵ_F^\pm are the new Fermi energies in the coexistence phase. However, to get a nonzero Δ from equation (5), one can have $E_k^\beta < 0$, which corresponds to the case $\epsilon_F^- > \epsilon_k > \epsilon_F^+$. Thus, the above self-consistent mean-field equations (4) and (5) at $T = 0$ take the form,

$$2\Delta_F = \lambda \int_{\epsilon_F^-}^{\epsilon_F^+} d\epsilon, \quad (6)$$

$$|\Delta| = g|\Delta| \left(\int_{-W/2}^{W/2} - \int_{\epsilon_F^-}^{\epsilon_F^+} \right) d\epsilon \frac{1}{\sqrt{(\epsilon - \mu)^2 + |\Delta|^2}}, \quad (7)$$

W being the band width. Equation (6) can be solved analytically and the FM order parameter is obtained as,

$$\Delta_F = \frac{\lambda}{\sqrt{\lambda^2 - 1}} |\Delta|. \quad (8)$$

Now, the SC order parameter from equation (7) can be computed by assuming the standard procedure of integration in a shell ($\Lambda/2$) around ϵ_F . In this approximation, equation (7) reduces to,

$$\frac{1}{g} = \left(\int_{\epsilon_F - \Lambda/2}^{\epsilon_F + \Lambda/2} - \int_{\epsilon_F^-}^{\epsilon_F^+} \right) d\epsilon \frac{1}{\sqrt{(\epsilon - \mu)^2 + |\Delta|^2}}. \quad (9)$$

where $\epsilon_F + \Lambda/2 > \epsilon_F^+$ and $\epsilon_F - \Lambda/2 < \epsilon_F^-$. Performing the integration and substituting Δ_F from equation (8), one obtains,

$$|\Delta| = \sqrt{\frac{\lambda - 1}{\lambda + 1}} \Lambda e^{-1/g}. \quad (10)$$

Thus, Δ_F can be calculated from equation (8) as,

$$\Delta_F = \frac{\lambda}{\lambda + 1} \Lambda e^{-1/g}. \quad (11)$$

Furthermore, putting the above values of Δ_F and Δ in the expression for ϵ_F^\pm , one obtains,

$$\epsilon_F^\pm = \epsilon_F \pm \frac{1}{\lambda + 1} \Lambda e^{-1/g}. \quad (12)$$

The above equations (10), (11) and (12) are of crucial importance in the present manuscript. It is clear from these equations that the SC gap Δ as well as the uniform magnetization ($\propto \Delta_F$) decrease as one approaches the Stoner threshold ($\lambda = 1$). Δ vanishes exactly at $\lambda = 1$ whereas Δ_F doesn't. This is an indication that the SC pairing induces spontaneous magnetization in the system which does not vanish at Stoner threshold (In principle $\Delta_F = 0$ at Stoner threshold). Thus, the para-Ferro QCP becomes a hidden one due to the presence of SC pairing. Furthermore, the new Fermi energy ϵ_F^\pm moves away more and more from ϵ_F , as one approaches the Stoner point. It becomes exactly equal to the Fermi energy of the Stoner FM ($\epsilon_F^\pm = \epsilon_F \pm \Delta_F$) at the Stoner threshold.

Next, let us consider the distribution functions n_k^\uparrow and n_k^\downarrow for the spin up and spin down quasi particles in terms of the Bogoliubov Fermions. These are already discussed in an earlier literature [4] and for completeness we can write them as,

$$n_k^\uparrow = u_k^2 n_k^\alpha + v_k^2 n_k^\beta = v_k^2 [\theta(k_F^- - k) + \theta(k - k_F^+)], \quad (13)$$

$$n_k^\downarrow = 1 - u_k^2 n_k^\beta - v_k^2 n_k^\alpha = n_k^\uparrow + [\theta(k_F^+ - k) - \theta(k_F^- - k)], \quad (14)$$

where u_k^2 and v_k^2 are the coherence factors involved in the Bogoliubov transformation which have the standard form in any mean-field theory. As already discussed before, at $T = 0$, $n_k^\alpha = 0$ and $n_k^\beta = \theta(k_F^- - k) + \theta(k - k_F^+)$. k_F^\pm are the wave vectors corresponding to the new Fermi energy ϵ_F^\pm . It should be noted at this point that, in a standard SC theory, a gap appears around the Fermi surface, but in the present case, Fermi surfaces appear for the Bogoliubov Fermion β in the coexistence phase which is completely unexpected. This could be due to the fact that the itinerant FM had already have the Fermi surfaces which still persist in the coexistence phase. Therefore, the existence of two Fermi surfaces is a generic property of the coexistence phase of both FM and SC since it is caused by the same quasi particles in the system. These Fermi surfaces are already reflected in the spin up and down momentum distribution functions and will lead to different properties in the system as compared to a standard mean-field theory. The single particle density of states which appears in almost all the properties of the system turns out to be,

$$N(0) = \frac{\rho(0)(\epsilon_F^+ + \epsilon_F^-)\Delta_F}{2\epsilon_F\sqrt{\Delta_F^2 - |\Delta|^2}} = N^+(0) + N^-(0), \quad (15)$$

where $N^+(0)$ and $N^-(0)$ are respectively the density of states on the two Fermi surfaces ϵ_F^+ and ϵ_F^- of the Bogoliubov Fermion β . For finite Δ_F , the density of states

increases with λ , as opposed to the case of a standard FM metal. The presence of Fermi surfaces together with the enhanced density of states at the Fermi level have important consequences in the thermodynamic properties of the system. The specific heat capacity, for example, at low temperature shows linear temperature dependence ($C_v(T) = \gamma T$) as opposed to the activated behaviour. This can again be understood in terms of the presence of Fermi surfaces of the β -Fermion. Moreover, the γ -coefficient in the specific heat which depends on $N(0)$ also gets enhanced due to increase in the density of states.

The increase in the single particle density of states can be understood in the following way: One can investigate the changes in the energy dispersion of the β -Fermion due to the appearance of the new Fermi energy in the coexistence phase. Substituting the expression for ϵ_F^\pm in E_k^β and approximating $\frac{\epsilon_k - \epsilon_F}{\Delta_F} \ll 1$, one can obtain the energy dispersion for the β -fermion as,

$$E_k^\beta \approx \pm \frac{\sqrt{\Delta_F^2 - \Delta^2}}{\Delta_F} (\epsilon_k - \epsilon_F), \quad (16)$$

which is just the renormalized free Fermion dispersion. The renormalization factor $\frac{\sqrt{\Delta_F^2 - \Delta^2}}{\Delta_F}$ enters not only in the energy dispersion but also in the density of states which is obvious from equation (15) and (16). Thus, the enhancement in the density of states at the Fermi level can be thought to be due to the reduction in the band width. This can also cause an increase in the effective mass ($m^* = \frac{m\Delta_F}{\sqrt{\Delta_F^2 - \Delta^2}}$) similar to that of density of states. However, the enhancement in the density of states/effective mass or the reduction in the β -Fermion band becomes prominent when one moves away from the para-Ferro QCP. This is due to the fact that the renormalization factor becomes unity at the Stoner point.

Let us now consider the effect of induced magnetization due to SC pairing in the spin wave dispersion. This can be achieved by analyzing the RPA transverse susceptibility [6] in the coexistence phase, which is given as,

$$\chi_{RPA}^{+-}(q, \omega) = \frac{\chi_0^{+-}(q, \omega)}{1 - U\chi_0^{+-}(q, \omega)}, \quad (17)$$

where $\chi_0^{+-}(q, \omega)$ is the unperturbed transverse susceptibility in the coexistence phase. Using the expressions for the Bogoliubov coherence factors, it can be computed as,

$$\begin{aligned} \chi_0^{+-}(q, \omega) = & \frac{1}{4} \sum_k \left(1 - \frac{\epsilon_k \epsilon_{k+q} + \Delta^2}{E_k E_{k+q}} \right) \\ & \left(\frac{1}{\omega + 2\Delta_F + E_{k+q} + E_k} + \frac{1}{\omega + 2\Delta_F - E_{k+q} - E_k} \right) \\ & + \frac{1}{4} \sum_k \left(1 + \frac{\epsilon_k \epsilon_{k+q} + \Delta^2}{E_k E_{k+q}} \right) \\ & \left(\frac{1}{\omega + 2\Delta_F + E_{k+q} - E_k} + \frac{1}{\omega + 2\Delta_F - E_{k+q} + E_k} \right). \end{aligned} \quad (18)$$

Spin wave dispersion can be obtained from the divergence of $\chi_{RPA}^{+-}(q, \omega)$, i. e., from the solutions of the equation $1 - U\chi_0^{+-}(q, \omega) = 0$. Expanding $\chi_0^{+-}(q, \omega)$ for small q and ω and for $\frac{\omega}{2\Delta_F} \ll 1$, the spin wave dispersion turns out to be,

$$\omega = Dq^2, \quad (19)$$

where the spin wave stiffness D is computed as, $D = \frac{1}{18} \frac{\Delta_F \sqrt{\Delta_F^2 - \Delta^2}}{\epsilon_F^2 m}$, m being the bare electron mass. The spin wave stiffness is reduced compared to that in the pure FM phase and becomes finite even at the Stoner critical point. This is due to the fact that the induced magnetization caused by SC pairing in the coexistence phase remains finite at the Stoner critical point.

Another important feature of the coexistence phase is the appearance of Fermi surfaces in the system. The consequence of this is the presence of paramagnons which describe the longitudinal spin fluctuations [6]. They not only survive in the FM metallic phase but also in the coexistence phase of both FM and SC. The propagator for the longitudinal spin fluctuations is given as,

$$\chi_l(q, \omega) = \frac{1}{\eta + bq^2 + \frac{ic|\omega|}{q}}, \quad (20)$$

where b and c are constants depending on the parameters in the system and η , which is the inverse of the static susceptibility is given by,

$$\eta = 1 - UN(0). \quad (21)$$

As we have already mentioned earlier, the density of states equation (15), increases with λ which makes the inverse of the static susceptibility η to vanish even if for small Δ_F . This is quite different from that of weak FM metals where η becomes zero at zero magnetization. Thus, the finite value of induced magnetization makes the Stoner QCP hidden.

The results obtained for the coexistence phase in the present manuscript is described only in the mean-field level which becomes a starting point for going beyond it. Since the appearance of induced magnetization in the coexistence phase makes the para-Ferro QCP a hidden one, it would be important to investigate the role of quantum fluctuations on it which is left for future study [7]. The conclusion that the para-Ferro QCP becomes a hidden one has also been pointed out recently in case of the coexistence of FM and spin triplet SC [8]. Thus, one can conclude that the hidden QCP might be a generic property of the coexistence phase where both the spin rotational and the gauge symmetry are broken and is independent of the symmetry of the SC order parameter.

However, in the present work, the SC QCP is dressed in the sense that SC can occur at zero magnetization. This is due to the fact that the SC pairing is caused not by spin fluctuations rather by some other means such as phonons. This could be contrasted with the standard spin fluctuation theory in an itinerant FM [9] where the QCP is naked. In the later case, the FM-SC transition temperature vanishes at the QCP. From the above scenarios, it might be possible to differentiate whether the SC in a FM is due to spin fluctuations or by some other means. The materials about which we mentioned at the beginning of the present manuscript fall into the second category where both SC and FM transition temperature vanish at the QCP. Thus, the SC mechanism in these materials might be thought to be due to spin fluctuations.

In conclusion, we briefly outline our findings. We consider a possible coexistence phase of both FM and SC. We solve the self-consistent mean-field equations for the uniform magnetization as well as the SC order parameter. It has been shown that both the order parameter decrease as one approaches the Stoner critical point. The SC gap vanishes exactly for $\lambda = 1$ but on the contrary, the uniform magnetization doesn't. This shows that the SC pairing induces a finite nonzero magnetization in the coexistence phase which washes out the Stoner QCP and makes it hidden. Moreover, we computed the effective mass as well as the spin wave dispersion in the coexistence phase. The former is enhanced but the later gets reduced and remains finite even at the Stoner threshold. Furthermore, the Bogoliubov Fermions in the coexistence phase retains the Fermi surfaces, which gets reflected in the thermodynamic properties of the system. In particular, the specific heat capacity has linear temperature dependence as in the standard itinerant FM, but the

γ -coefficient increases anomalously for a small magnetization. This is due to the fact that the single particle density of state in the coexistence phase gets enhanced.

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